Greek, Arabic, and Magic Squares

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About me

- Serpent Herald
- Former Palimpsest
- College classes in Greek, Latin, medieval Near Eastern history
- Math PhD
- Maintain the Medieval Names Archive
- Companion of the Order of the Laurel

Where can you find these slides?

- At `https://tinyurl.com/MagicSquares2024`
- At my website, `www.yarntheory.net/ursulageorges/` (eventually)

Some class components

- Addition and multiplication
- Modern algebraic notation
- Ancient Greek letters and numbers
- Arabic letters and numbers
- Medieval ideas about magic

What's magic about the math of this square?

2	7	6
9	5	1
4	3	8

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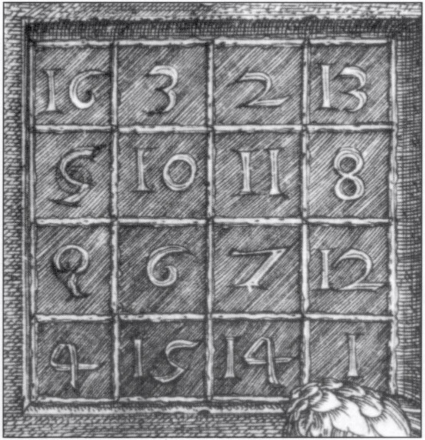
A definition

An $n \times n$ magic square

- Has $n \cdot n = n^2$ different positive numbers arranged in a square
- The sums along the rows, columns, and diagonals are all the same.

Example: Dürer

Albrecht Dürer (1471–1528), *Melencolia I*:



Example: Cheng Dawei

Cheng Dawei (1533–1606), *Suanfa Tongzong*:

Our key source

Jacques Sesiano, A treatise on the use of magic squares

- *Suhayl* 19 (2022), 89–150.
- Analyzes a 1675 copy of a 13th- or 14th-century Arabic manuscript.

Are you basic?

- A basic magic square uses the numbers $1, 2, 3, \ldots, n^2$.
- The smallest basic magic square is the 3×3 square with sum 15.

Basic sums

Side length n	Basic sum
3	15
4	34
5	65
$\vdots$	$\vdots$
n	$\frac{n(n^2+1)}{2}$

Our source's description of the basic sum

One multiplies the (number of) cells on the side of the figure by itself; the result will be the quantity of all cells of the figure. Adding 1 to it gives the equating number. Consider now the equating number. If it is even, one multiplies its half by the total number of cells on one side, and if it is odd, one multiplies the whole of it by half the number of cells on the side; the result will be the basic magic sum for one row.

First goals

- How do you make a basic 3×3 magic square?
- How do you make a 3×3 magic square with a different sum?

Some medieval instructions

(The method) is the following. 1 is put in the middle cell of the first column, 2 in the corresponding (lower) knight's cell, 3 in the knight's cell of the latter, 4 next to it above (which is the knight's (cell) of 1), 5 in its queen's cell, 6 likewise, 7 next to it, 8 in its knight's cell, 9 likewise. It is possible to have the beginning in any middle (side cell), the remainder (of the placing) being as you know.

The resulting square

8	3	4
1	5	9
6	7	2

New squares from old

Question

Can we use the 3×3 magic squares we have already seen to make new magic squares?

Adding the same number

If we add the same number to every row in a 3×3 magic square

. . .

Adding the same number

If we add the same number to every row in a 3×3 magic square . . .

- We get a new magic sum.
- The new magic sum is a multiple of 3.

Practice

Let's make a magic square with magic sum 27.

The magic of letters

Numbers $\longleftrightarrow$ Greek letters $\longleftrightarrow$ Arabic letters

Greek numerals

- Greek letters can be used as numbers
- Idea: 1st nine letters are 1, . . . , 9, 2nd nine letters are 10, . . . , 90, 3rd nine letters are 100, . . . , 900
- Use a bar over the letters to indicate they are acting as numerals.

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Example

$\bar{\alpha} = 1$ and $\bar{\lambda} = 30$. What is $\overline{\lambda\alpha}$?

Alphabets change!

- The Greek alphabet used since the 4th century BCE only has 24 letters
- We need $3 \cdot 9 = 27$ symbols
- Solution: Use archaic letters!

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Greek magic square practice

Let's make a magic square representing $\phi\iota\lambda$ (Phil.)

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Moving to Arabic

Problem

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Solution

Match the final letter (غ or ghayn) to 1000.

Very basic facts about Arabic

- Reads from right to left.
- Letters are consonants (or long vowels)
- Short vowels are (optional) decorations

An Arabic magic square

Let's make a magic square for the Arabic malik مَلِك, 'king'.

م+ل+ك

Other magic numbers?

How should we handle sums that are not a multiple of 3?

Consecutive triples

- Divide 1, . . . , 9 into three consecutive triples: 1,2,3; 4,5,6; 7,8,9.
- Notice that the numbers in each triple are in different rows and columns of our magic square.

Magic 32

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- Add 1 to members of the final 2 series
- Add 15 to all the numbers!

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